



انجمن منطق ایران

First-Order Interpolation Derived from Propositional Interpolation

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Abstract:

Ever since Craig's seminal paper on interpolation [3], interpolation properties have been recognized as important properties of logical systems. Recall that a logic L has *interpolation* if whenever $A \rightarrow B$ holds in L there exists a formula I in the common language of A and B such that $A \rightarrow I$ and $I \rightarrow B$ hold in L .

Propositional interpolation properties can be determined and classified with relative ease using the ground-breaking results of Maksimova cf. [7, 6, 5]. This approach is based on an algebraic analysis of the logic in question. In contrast first-order interpolation properties are notoriously hard to determine, even for logics where propositional interpolation is more or less obvious. For example it is unknown whether $G_{[0,1]}^{QF}$ (first-order infinitely-valued Gödel logic) interpolates (cf. [1]) and even for MC^{QF} , the logic of constant domain Kripke frames of three worlds with two top worlds (an extension of MC), interpolation proofs are very hard cf. Ono [8]. This situation is due to the lack of an adequate algebraization of non-classical first-order logics. In this paper we present a proof theoretic methodology to reduce first-order interpolation to propositional interpolation:

$$\left. \begin{array}{l} \text{existence of suitable skolemizations} + \\ \text{existence of Herbrand expansions} + \\ \text{propositional interpolation} \end{array} \right\} \Rightarrow \text{first-order interpolation.}$$

The construction of the first-order interpolant from the propositional interpolant follows this procedure:

1. Develop a validity equivalent skolemization replacing all strong quantifiers³ in the valid formula $A \rightarrow B$ to obtain the valid formula $A_1 \rightarrow B_1$.
2. Construct a valid Herbrand expansion $A_2 \rightarrow B_2$ for $A_1 \rightarrow B_1$. Occurrences of $\exists xB(x)$ and $\forall xA(x)$ are replaced by suitable finite disjunctions $\bigvee B(t_i)$ and conjunctions $\bigwedge B(t_i)$, respectively.
3. Interpolate the propositionally valid formula $A_2 \rightarrow B_2$ with the propositional interpolant I^* : $A_2 \rightarrow I^*$ and $I^* \rightarrow B_2$ are propositionally valid.
4. Reintroduce weak quantifiers to obtain valid formulas $A_1 \rightarrow I^*$ and $I^* \rightarrow B_1$.
5. Eliminate all function symbols and constants not in the common language of A_1 and B_1 by introducing suitable quantifiers in I^* (note that no Skolem functions are in the common language, therefore they are eliminated). Let I be the result.
6. I is an interpolant for $A_1 \rightarrow B_1$. $A_1 \rightarrow I$ and $I \rightarrow B_1$ are skolemizations of $A \rightarrow I$ and $I \rightarrow B$. Therefore I is an interpolant of $A \rightarrow B$.

It is decidable if propositional lattice based finitely-values logics admit the interpolation property [2]. Consequently, it is decidable if finitely-valued first-order logics admit the interpolation property. In this paper we extend the methodology to prenex fragments where Skolemization is admissible due to the second epsilon theorem [4].

References

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